Significant Figures in Measurements

Supermarkets often provide scales like the one in Figure 3.4. Customers use these scales to measure the weight of produce that is priced per pound. If you use a scale that is calibrated in 0.1-lb intervals, you can easily read the scale to the nearest tenth of a pound. With such a scale, however, you can also estimate the weight to the nearest hundredth of a pound by noting the position of the pointer between calibration marks.

Suppose you estimate a weight that lies between 2.4 lb and 2.5 lb to be 2.46 lb. The number in this estimated measurement has three digits. The first two digits in the measurement (2 and 4) are known with certainty. But the rightmost digit (6) has been estimated and involves some uncertainty. These three reported digits all convey useful information, however, and are called significant figures. The **significant figures** in a measurement include all of the digits that are known, plus a last digit that is estimated. **Measurements must always be reported to the correct number of significant figures because calculated answers often depend on the number of significant figures in the values used in the calculation.**

Instruments differ in the number of significant figures that can be obtained from their use and thus in the precision of measurements. The three meter sticks in Figure 3.5 can be used to make successively more precise measurements of the board.

Figure 3.4 The precision of a weighing scale depends on how finely it is calibrated.

Interactive Textbook

Animation 2 See how the precision of a calculated result depends on the sensitivity of the measuring instruments.

with ChemASAP

Rules for determining whether a digit in a measured value is significant:

1 Every nonzero digit in a reported measurement is assumed to be significant. The measurements 24.7 meters, 0.743 meter, and 714 meters each express a measure of length to three significant figures.

2 Zeros appearing between nonzero digits are significant. The measurements 7003 meters, 40.79 meters, and 1.503 meters each have four significant figures.

3 Leftmost zeros appearing in front of nonzero digits are not significant. They act as placeholders. The measurements 0.0071 meter, 0.42 meter, and 0.000 099 meter each have only two significant figures. The zeros to the left are not significant. By writing the measurements in scientific notation, you can eliminate such placeholding zeros: in this case, 7.1×10^{-3} meter, 4.2×10^{-1} meter, and 9.9×10^{-5} meter.

4 Zeros at the end of a number and to the right of a decimal point are always significant. The measurements 43.00 meters, 1.010 meters, and 9.000 meters each have four significant figures.

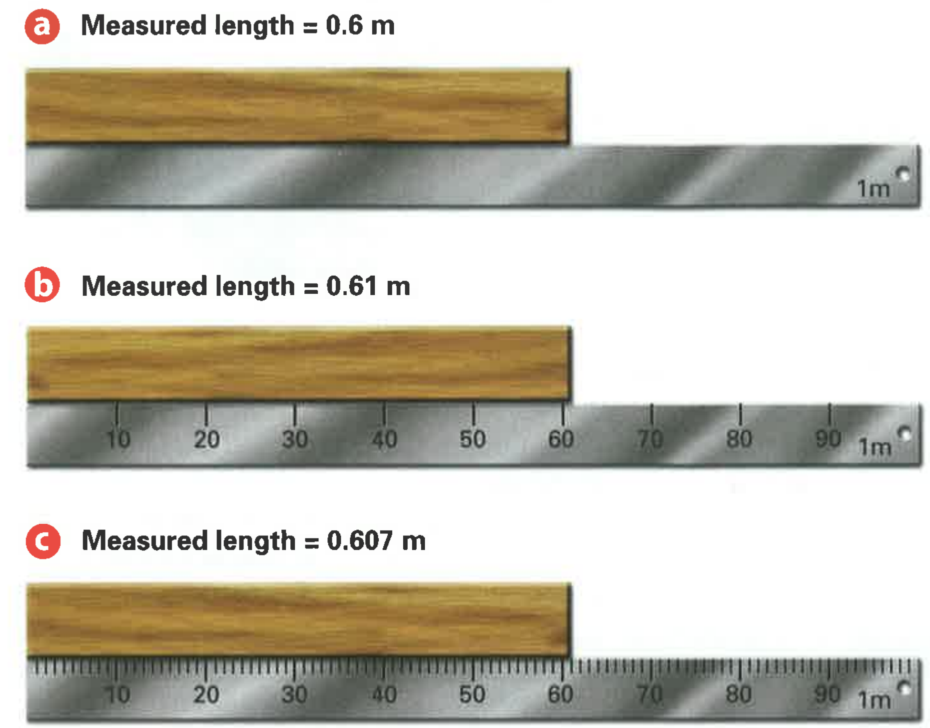
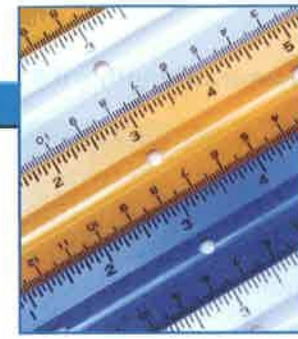


Figure 3.5 Three differently calibrated meter sticks are used to measure the length of a board. **a** A meter stick calibrated in a 1-m interval. **b** A meter stick calibrated in 0.1-m intervals. **c** A meter stick calibrated in 0.01-m intervals. **Measuring** ***How many significant figures are reported in each measurement?***

5 Zeros at the rightmost end of a measurement that lie to the left of an understood decimal point are not significant if they serve as placeholders to show the magnitude of the number. The zeros in the measurements 300 meters, 7000 meters, and 27,210 meters are not significant. The numbers of significant figures in these values are one, one, and four, respectively. If such zeros were known measured values, however, then they would be significant. For example, if all of the zeros in the measurement 300 meters were significant, writing the value in scientific notation as 3.00×10^2 meters makes it clear that these zeros are significant.

6 There are two situations in which numbers have an unlimited number of significant figures. The first involves counting. If you count 23 people in your classroom, then there are exactly 23 people, and this value has an unlimited number of significant figures. The second situation involves exactly defined quantities such as those found within a system of measurement. When, for example, you write 60 min = 1 hr, or 100 cm = 1 m, each of these numbers has an unlimited number of significant figures. As you shall soon see, exact quantities do not affect the process of rounding an answer to the correct number of significant figures.

CONCEPTUAL PROBLEM 3.1



Counting Significant Figures in Measurements

How many significant figures are in each measurement?

a. 123 m
b. 40,506 mm
c. 9.8000×10^4 m
d. 22 meter sticks
e. 0.070 80 m
f. 98,000 m

1 Analyze ***Identify the relevant concepts.***

The location of each zero in the measurement and the location of the decimal point determine which of the rules apply for determining significant figures.

2 Solve ***Apply the concepts to this problem.***

All nonzero digits are significant (rule 1). Use rules 2 through 6 to determine if the zeros are significant.

a. three (rule 1)
b. five (rule 2)
c. five (rule 4)
d. unlimited (rule 6)
e. four (rules 2, 3, 4)
f. two (rule 5)

Practice Problems

1. Count the significant figures in each length.
a. 0.057 30 meters
b. 8765 meters
c. 0.000 73 meters
d. 40.007 meters

2. How many significant figures are in each measurement?
a. 143 grams
b. 0.074 meter
c. 8.750×10^{-2} gram
d. 1.072 meter

Interactive Textbook

Problem-Solving 3.2 Solve Problem 2 with the help of an interactive guided tutorial.

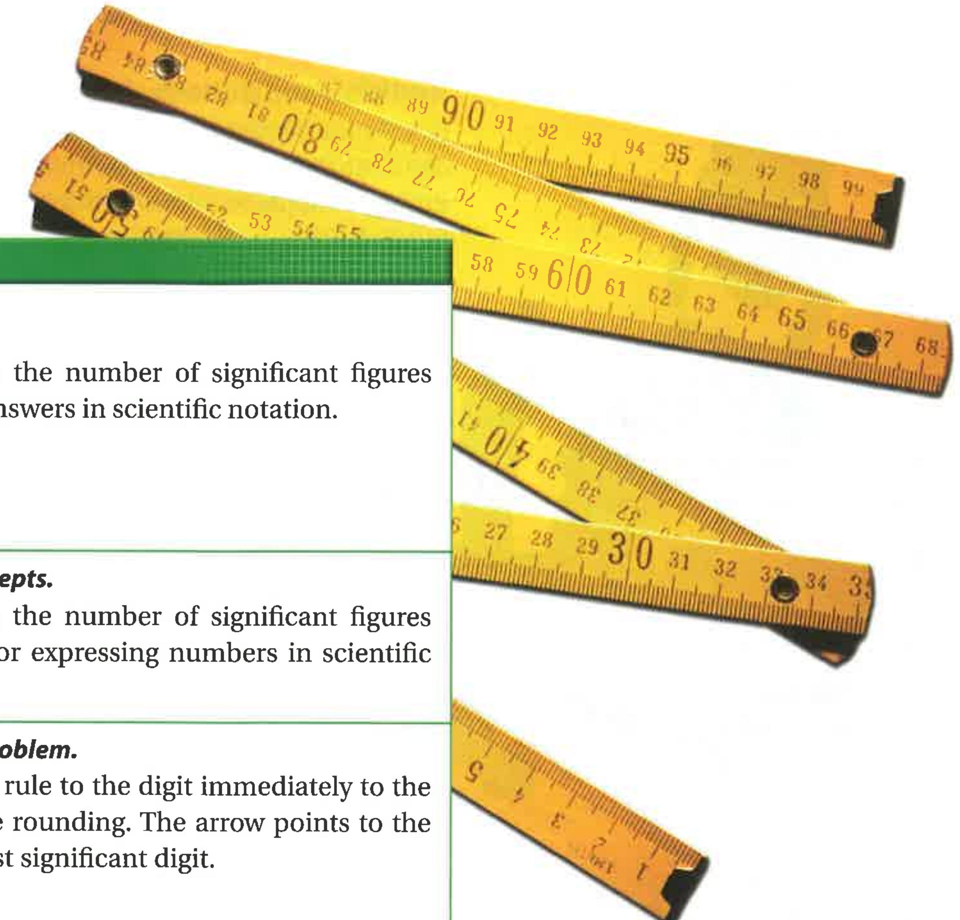
with ChemASAP

Significant Figures in Calculations

Suppose you use a calculator to find the area of a floor that measures 7.7 meters by 5.4 meters. The calculator would give an answer of 41.58 square meters. The calculated area is expressed to four significant figures. However, each of the measurements used in the calculation is expressed to only two significant figures. So the answer must also be reported to two significant figures (42 m^2). **In general, a calculated answer cannot be more precise than the least precise measurement from which it was calculated.** The calculated value must be rounded to make it consistent with the measurements from which it was calculated.

Rounding To round a number, you must first decide how many significant figures the answer should have. This decision depends on the given measurements and on the mathematical process used to arrive at the answer. Once you know the number of significant figures your answer should have, round to that many digits, counting from the left. If the digit immediately to the right of the last significant digit is less than 5, it is simply dropped and the value of the last significant digit stays the same. If the digit in question is 5 or greater, the value of the digit in the last significant place is increased by 1.

Checkpoint ***Why must a calculated answer generally be rounded?***



SAMPLE PROBLEM 3.1

Rounding Measurements

Round off each measurement to the number of significant figures shown in parentheses. Write the answers in scientific notation.

a. 314.721 meters (four)
b. 0.001 775 meter (two)
c. 8792 meters (two)

1 Analyze ***Identify the relevant concepts.***

Round off each measurement to the number of significant figures indicated. Then apply the rules for expressing numbers in scientific notation.

2 Solve ***Apply the concepts to this problem.***

Count from the left and apply the rule to the digit immediately to the right of the digit to which you are rounding. The arrow points to the digit immediately following the last significant digit.

a. 314.721 meters
↑
2 is less than 5, so you do not round up.
314.7 meters = 3.147×10^2 meters

b. 0.001 775 meter
↑
7 is greater than 5, so round up.
0.0018 meter = 1.8×10^{-3} meter

c. 8792 meters
↑
9 is greater than 5, so round up
8800 meters = 8.8×10^3 meters

3 Evaluate ***Do the results make sense?***

The rules for rounding and for writing numbers in scientific notation have been correctly applied.

Practice Problems

3. Round each measurement to three significant figures. Write your answers in scientific notation.
a. 87.073 meters
b. 4.3621×10^8 meters
c. 0.01552 meter
d. 9009 meters
e. 1.7777×10^{-3} meter
f. 629.55 meters

4. Round each measurement in Practice Problem 3 to one significant figure. Write each of your answers in scientific notation.

Math Handbook

For help with scientific notation, go to page R56.

Interactive Textbook

Problem-Solving 3.3 Solve Problem 3 with the help of an interactive guided tutorial.

with ChemASAP

Addition and Subtraction

The answer to an addition or subtraction calculation should be rounded to the same number of decimal places (not digits) as the measurement with the least number of decimal places. Work through Sample Problem 3.2 below which provides an example of rounding in an addition calculation.

SAMPLE PROBLEM 3.2

Significant Figures in Addition

Calculate the sum of the three measurements. Give the answer to the correct number of significant figures.

12.52 meters + 349.0 meters + 8.24 meters

1 Analyze ***Identify the relevant concepts.***
Calculate the sum and then analyze each measurement to determine the number of decimal places required in the answer.

2 Solve ***Apply the concepts to this problem.***
Align the decimal points and add the numbers. Round the answer to match the measurement with the least number of decimal places.

$$\begin{array}{rl} 12.52 & \text{meters} \\ 349.0 & \text{meters} \\ +\ \ 8.24 & \text{meters} \\ \hline 369.76 & \text{meters} \end{array}$$

The second measurement (349.0 meters) has the least number of digits (one) to the right of the decimal point. Thus the answer must be rounded to one digit after the decimal point. The answer is rounded to 369.8 meters, or 3.698×10^2 meters.

3 Evaluate ***Does the result make sense?***
The mathematical operation has been correctly carried out and the resulting answer is reported to the correct number of decimal places.

Practice Problems

5. Perform each operation. Express your answers to the correct number of significant figures.
- **a.** 61.2 meters + 9.35 meters + 8.6 meters
- **b.** 9.44 meters − 2.11 meters
- **c.** 1.36 meters + 10.17 meters
- **d.** 34.61 meters − 17.3 meters

6. Find the total mass of three diamonds that have masses of 14.2 grams, 8.73 grams, and 0.912 gram.

Math Handbook
For help with significant figures, go to page R59.

Interactive Textbook
Problem-Solving 3.6 Solve Problem 6 with the help of an interactive guided tutorial.
with ChemASAP

Multiplication and Division

In calculations involving multiplication and division, you need to round the answer to the same number of significant figures as the measurement with the least number of significant figures. The position of the decimal point has nothing to do with the rounding process when multiplying and dividing measurements. The position of the decimal point is important only in rounding the answers of addition or subtraction problems.

Checkpoint ***How many significant figures must you round an answer to when performing multiplication or division?***

SAMPLE PROBLEM 3.3

Significant Figures in Multiplication and Division

Perform the following operations. Give the answers to the correct number of significant figures.
- **a.** 7.55 meters × 0.34 meter
- **b.** 2.10 meters × 0.70 meter
- **c.** 2.4526 meters ÷ 8.4

1 Analyze ***Identify the relevant concepts.***
Perform the required math operation and then analyze each of the original numbers to determine the correct number of significant figures required in the answer.

2 Solve ***Apply the concepts to this problem.***
Round the answers to match the measurement with the least number of significant figures.
- **a.** 7.55 meters × 0.34 meter = 2.567 $(\text{meter})^2$ = 2.6 meters^2
  (0.34 meter has two significant figures)
- **b.** 2.10 meters × 0.70 meter = 1.47 $(\text{meter})^2$ = 1.5 meters^2
  (0.70 meter has two significant figures)
- **c.** 2.4526 meters ÷ 8.4 = 0.291 976 meter = 0.29 meter
  (8.4 has two significant figures)

3 Evaluate ***Do the results make sense?***
The mathematical operations have been performed correctly, and the resulting answers are reported to the correct number of places.

Practice Problems

7. Solve each problem. Give your answers to the correct number of significant figures and in scientific notation.
- **a.** 8.3 meters × 2.22 meters
- **b.** 8432 meters ÷ 12.5
- **c.** 35.2 seconds × $\frac{1 \text{ minute}}{60 \text{ seconds}}$

8. Calculate the volume of a warehouse that has inside dimensions of 22.4 meters by 11.3 meters by 5.2 meters. (Volume = $l \times w \times h$)

Math Handbook
For help with using a calculator, go to page R62.

Interactive Textbook
Problem-Solving 3.8 Solve Problem 8 with the help of an interactive guided tutorial.
with ChemASAP

Quick LAB

Accuracy and Precision

Purpose

To measure the dimensions of an object as accurately and precisely as possible and to apply rules for rounding answers calculated from the measurements.

Materials

- 3 inch × 5 inch index card
- metric ruler

Procedure

1. Use a metric ruler to measure in centimeters the length and width of an index card as accurately and precisely as you can. The hundredths place in your measurement should be estimated.
2. Calculate the perimeter [2 × (length + width)] and the area (length × width) of the index card. Write both your unrounded answers and your correctly rounded answers on the chalkboard.

Analyze and Conclude

1. How many significant figures are in your measurements of length and of width?
2. How do your measurements compare with those of your classmates?
3. How many significant figures are in your calculated value for the area? In your calculated value for the perimeter? Do your rounded answers have as many significant figures as your classmates' measurements?
4. Assume that the correct (accurate) length and width of the card are 12.70 cm and 7.62 cm, respectively. Calculate the percent error for each of your two measurements.

3.1 Section Assessment

9. **Key Concept** How do measurements relate to experimental science?
10. **Key Concept** How are accuracy and precision evaluated?
11. **Key Concept** Why must a given measurement always be reported to the correct number of significant figures?
12. **Key Concept** How does the precision of a calculated answer compare to the precision of the measurements used to obtain it?
13. A technician experimentally determined the boiling point of octane to be 124.1°C. The actual boiling point of octane is 125.7°C. Calculate the error and the percent error.
14. Determine the number of significant figures in each of the following.
    a. 11 soccer players **b.** 0.070 020 meter
    c. 10,800 meters **d.** 5.00 cubic meters
15. Solve the following and express each answer in scientific notation and to the correct number of significant figures.
    a. $(5.3 \times 10^4) + (1.3 \times 10^4)$
    b. $(7.2 \times 10^{-4}) \div (1.8 \times 10^3)$
    c. $10^4 \times 10^{-3} \times 10^6$
    d. $(9.12 \times 10^{-1}) - (4.7 \times 10^{-2})$
    e. $(5.4 \times 10^4) \times (3.5 \times 10^9)$

Writing Activity

Explanatory Paragraph Explain the differences between the accuracy, precision, and error of a measurement.

Interactive Textbook

Assessment 3.1 Test yourself on the concepts in Section 3.1.

with ChemASAP

3.2 The International System of Units

Connecting to Your World "Are we there yet?" You may have asked this question during a long road trip with family or friends. To find out how much farther you have to go, you can read the roadside signs that list destinations and their distances. In the signs shown here, however, the distances are listed as numbers with no units attached. Is Carrieton 44 kilometers or 44 miles away? Without the units, you can't be sure. When you make a measurement, you must assign the correct units to the numerical value. Without the units, it is impossible to communicate the measurement clearly to others.

Guide for Reading

Key Concepts

- Which five SI base units do chemists commonly use?
- What metric units are commonly used to measure length, volume, mass, temperature, and energy?

Vocabulary

International System of Units (SI)
meter (m)
liter (L)
kilogram (kg)
gram (g)
weight
temperature
Celsius scale
Kelvin scale
absolute zero
energy
joule (J)
calorie (cal)

Reading Strategy

Summarizing As you read about SI units, summarize the main ideas in the text that follows the red and blue headings.

Measuring with SI Units

All measurements depend on units that serve as reference standards. The standards of measurement used in science are those of the metric system. The metric system is important because of its simplicity and ease of use. All metric units are based on multiples of 10. As a result, you can convert between units easily. The metric system was originally established in France in 1795. The **International System of Units** (abbreviated **SI**, after the French name, *Le Système International d'Unités*) is a revised version of the metric system. The SI was adopted by international agreement in 1960. There are seven SI base units, which are listed in Table 3.1. From these base units, all other SI units of measurement can be derived. **The five SI base units commonly used by chemists are the meter, the kilogram, the kelvin, the second, and the mole.**

All measured quantities can be reported in SI units. Sometimes, however, non-SI units are preferred for convenience or for practical reasons. In this textbook you will learn about both SI and non-SI units.

Table 3.1

SI Base Units

Quantity	SI base unit	Symbol
Length	meter	m
Mass	kilogram	kg
Temperature	kelvin	K
Time	second	s
Amount of substance	mole	mol
Luminous intensity	candela	cd
Electric current	ampere	A