10 Feeling Under Pressure
Boyle's Law

Think About It
A flat basketball does not bounce very well. However, you can push air into the ball with a pump. The air you trap inside pushes on the skin of the ball with a lot of pressure. The amount of air you push into the ball determines how firm and bouncy the ball is. What would happen if you pushed the same amount of air into a smaller ball?

How does gas volume affect gas pressure?
To answer this question, you will explore
1. The Syringe and Scale
2. Graphing Pressure–Volume Data
3. Boyle's Law

Exploring the Topic
1. The Syringe and Scale
In class you examined how the pressure of a gas trapped inside a container is related to the volume the gas occupies. A syringe makes a good container for such an investigation, because it can be sealed off and the volume of air trapped inside the syringe can be measured.

If you push down on the plunger of the syringe, you can feel the pressure of the gas inside the syringe. Air molecules take up space and as you squeeze them into a smaller space, they push back more and more. This is because the same number of molecules occupies a smaller volume, resulting in more collisions between the molecules and the container. More collisions mean greater gas pressure.
Gathering Data

You can learn more about the relationship between the volume of a gas and its pressure by using a capped syringe and a bathroom scale. As you push down on the plunger of the syringe, the gas pressure inside the syringe increases, and the weight on the scale increases. You can record this quantitative, or numerical, data. To calculate the pressure of the gas, divide the measured weight on the scale by the surface area of the plunger in pounds per square inch, or lb/in². Weight is a measure of force.

Pressure applied = weight
That you apply divided by area,
plus atmospheric pressure.

Area of the plunger in contact
with the gas is the cross-sectional
area inside the plunger.

Note that the pressure you apply plus atmospheric pressure is equal to the gas pressure inside the syringe. When you push down, the gas pushes back with the same pressure that is being applied to it.

**Important** You need to add 14.7 lb/in² to the pressure that you apply because the atmosphere is also pushing on the syringe.

Data for this experiment are given in the table. Notice that as the volume decreases, the pressure of the gas inside the syringe increases.

**Pressure and Volume Data**

<table>
<thead>
<tr>
<th>Trial</th>
<th>Volume (mL)</th>
<th>Total pressure (lb/in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>38</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>75</td>
</tr>
</tbody>
</table>

**BIG IDEA** If you squeeze a sample of gas into a smaller container, its pressure will increase.

**Biology Connection**

During takeoff and landing on an airplane, or at other times when your altitude changes, you may feel the need to "pop" your ears. Your middle ear contains a pocket of air. A tube that leads from the back of the nose to the ear supplies this air. When you swallow, air goes through the tube and into the pocket, so that the pressure inside your ear is the same as the air pressure. You experience the change in pressure for the gas inside your ear to match a change in air pressure as a "pop."

**Industry Connection**

Gases are often stored in tanks at high pressure or even at low temperatures so that they’re liquid, because otherwise the containers needed for them would be huge. These tanks are usually made of thick metal so that they don’t change volume.

**Graphing Pressure-Volume Data**

Many of the relationships you have explored so far, like mass versus volume and volume versus temperature of a gas, are proportional relationships. The relationship between pressure and volume is not directly proportional. A graph of the data for this experiment results in a curved line that does not go through the origin.

Gas pressure and gas volume are inversely proportional to one another. In other words, when one variable increases, the other decreases, and vice versa. When the volume of the gas in the syringe is very small, the gas pressure is quite high. When the volume of the gas is large, the gas pressure is quite low. However, as long as there are gas molecules in a container, neither the gas pressure nor the gas volume can ever reach zero.

**Boyle’s Law**

In 1662, a British scientist named Robert Boyle discovered that gas pressure and volume are inversely proportional to each other. This relationship is known as Boyle’s law.

**Boyle’s Law**

The pressure $P$ of a given amount of gas is inversely proportional to its volume $V$, if the temperature and amount of gas are not changed. The relationship is $PV = k$, or $P = k/V$, where $k$ is the proportionality constant.

Examine the Pressure and Volume Data table. If you multiply pressure by volume, the product will always be around 1500. (It will vary slightly due to rounding and slight errors in measurement.) Boyle’s law expresses this relationship as $PV = k$. In this experiment the proportionality constant, $k$, is 1500 mL - lb/in². For a review of this math topic, see MATH SPOTLIGHT: RATIO AND PROPORTIONS on page 627. Boyle’s law can be used to solve problems involving gas pressure and gas volume. Once you have one set of pressure and volume measurements for a gas, you can determine the proportionality constant. With the proportionality constant, you can determine the pressure of the gas at any volume.
Boyle's Law

Robert Boyle (1627–1691), an Irish chemist, did experiments like the one shown in Figure 14-2 to study the relationship between the pressure and the volume of a gas. By taking careful quantitative measurements, he showed that if the temperature is constant, doubling the pressure of a fixed amount of gas decreases its volume by one-half. On the other hand, reducing the pressure by half results in a doubling of the volume. A relationship in which one variable increases as the other variable decreases is referred to as an inversely proportional relationship. For help with understanding inverse relationships, see the Math Handbook page 905.

Boyle's law states that the volume of a given amount of gas held at a constant temperature varies inversely with the pressure. Look at the graph in Figure 14-2 in which pressure versus volume is plotted for a gas. The plot of an inversely proportional relationship results in a downward curve. If you choose any two points along the curve and multiply the pressure times the volume at each point, how do your two answers compare? Note that the product of the pressure and the volume for each of points 1, 2, and 3 is 10 atm·L. From the graph, what would the volume be if the pressure is 2.5 atm? What would the pressure be if the volume is 2 L?

The products of pressure times volume for any two sets of conditions are equal, so Boyle's law can be expressed mathematically as follows.

\[ P_1V_1 = P_2V_2 \]

\( P_1 \) and \( V_1 \) represent a set of initial conditions for a gas and \( P_2 \) and \( V_2 \) represent a set of new conditions. If you know any three of these four values for a gas at constant temperature, you can solve for the fourth by rearranging the equation. For example, if \( P_1 \), \( V_1 \), and \( P_2 \) are known, dividing both sides of the equation by \( P_2 \) will isolate the unknown variable \( V_2 \).

Use the equation for Boyle's law to calculate the volume that corresponds to a pressure of 2.5 atm, assuming that the amount of gas and temperature are constant. Then find what pressure corresponds to a volume of 2.0 L. Use 2.0 atm for \( P_1 \) and 5 L for \( V_1 \). How do these answers compare to those you found using the graph in Figure 14-2?

Figure 14-2

The gas particles in this cylinder take up a given volume at a given pressure. As pressure increases, volume decreases. The graph shows that pressure and volume have an inverse relationship, which means that as pressure increases, volume decreases. This relationship is illustrated by a downward curve in the line from condition 1 to condition 2 to condition 3.

Pressure–Volume Changes

<table>
<thead>
<tr>
<th>Condition 1:</th>
<th>Condition 2:</th>
<th>Condition 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k = \frac{P_1V_1}{10 L} )</td>
<td>( k = \frac{P_2V_2}{5 L} )</td>
<td>( k = \frac{P_3V_3}{2.5 L} )</td>
</tr>
<tr>
<td>( k = \frac{1 \text{ atm} \times 10 \text{ L}}{10 \text{ atm} \cdot \text{L}} )</td>
<td>( k = \frac{2 \text{ atm} \times 5 \text{ L}}{10 \text{ atm} \cdot \text{L}} )</td>
<td>( k = \frac{4 \text{ atm} \times 2.5 \text{ L}}{10 \text{ atm} \cdot \text{L}} )</td>
</tr>
</tbody>
</table>
EXAMPLE PROBLEM 14-1

Boyle's Law
A sample of helium gas in a balloon is compressed from 4.0 L to 2.5 L at a constant temperature. If the pressure of the gas in the 4.0-L volume is 210 kPa, what will the pressure be at 2.5 L?

1. Analyze the Problem

You are given the initial and final volumes and the initial pressure of a sample of helium. Boyle's law states that as volume decreases, pressure increases if temperature remains constant. Because the volume in this problem is decreasing, the pressure will increase. So the initial pressure should be multiplied by a volume ratio greater than one.

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1 = 4.0 \text{ L}$</td>
<td>$P_2 = ? \text{ kPa}$</td>
</tr>
<tr>
<td>$V_2 = 2.5 \text{ L}$</td>
<td>$P_1 = 210 \text{ kPa}$</td>
</tr>
</tbody>
</table>

2. Solve for the Unknown

Divide both sides of the equation for Boyle's law by $V_2$ to solve for $P_2$.

\[ P_1 \frac{V_1}{V_2} = P_2 \]

Substitute the known values into the rearranged equation.

\[ P_2 = P_1 \left( \frac{V_1}{V_2} \right) \]

Multiply and divide numbers and units to solve for $P_2$.

\[ P_2 = 210 \text{ kPa} \left( \frac{4.0 \text{ L}}{2.5 \text{ L}} \right) = 340 \text{ kPa} \]

3. Evaluate the Answer

When the volume is decreased by almost half, the pressure is expected to almost double. The calculated value of 340 kPa is reasonable. The unit in the answer is kPa, a pressure unit.

PRACTICE PROBLEMS

Assume that the temperature and the amount of gas present are constant in the following problems.

1. The volume of a gas at 99.0 kPa is 300.0 mL. If the pressure is increased to 188 kPa, what will be the new volume?

2. The pressure of a sample of helium in a 1.00-L container is 0.989 atm. What is the new pressure if the sample is placed in a 2.00-L container?

3. Air trapped in a cylinder fitted with a piston occupies 145.7 mL at 1.08 atm pressure. What is the new volume of air when the pressure is increased to 1.43 atm by applying force to the piston?

4. If it takes 0.0050 L of oxygen gas kept in a cylinder under pressure to fill an evacuated 4.00-L reaction vessel in which the pressure is 0.980 atm, what was the initial pressure of the gas in the cylinder?

5. A sample of neon gas occupies 0.220 L at 0.860 atm. What will be its volume at 29.2 kPa pressure?